Reducing Proof: Farmar's Spirit Rule and Some Antecedents

Greg Miller and Tom Martin

Abstract

Reducing the strength of a spirituous liquor by addition of water is a normal operation for distillers, blenders, and bottlers. Quantitatively predicting how much water is required for a given reduction has always been a challenge. Scientific developments in the 1790s resulted in the creation of slide rules that contained the necessary information, though the information was not organized in a helpful way. It was not until 1901 that Farmar's spirit slide rule was created with the specific intent of calculating the water needed for a reduction. Several slide rules that can be related to reduction, dating from ca. 1790 to ca. 1920, are described.

Introduction

Since 1868 "proof spirit" has been defined in the US as a mixture of water and pure alcohol such that the volume of alcohol is 50% of the total at 60°F,[1] and the US 0–200 proof scale is twice the 0–100 percentage alcohol by volume (ABV) scale. With this alcohol-volume-based understanding, the conservation of alcohol holds that volume V_1 at proof measure P_1 could be reduced to a lower proof measure P_2 with a different volume V_2, and that these quantities are related through the relation

$$V_1 P_1 = V_2 P_2. \qquad (1)$$

The UK Imperial proof, though differently defined, is also proportional to alcohol by volume, and is equally compatible with this formula. This elementary calculation could be easily done using any logarithmic slide rule, yet specialty slide rules were produced from ca. 1820 to perform this calculation with one scale labeled "proof" and the other for indicating volume and using an inverse scale (Fig. 1). Such "comparative" slide rules have been described elsewhere.[2] Martin,[3] commenting on the application of these slide rules to the problem of proof reduction, correctly notes that the volume of water to add is not $V_2 - V_1$, because when water and alcohol mix there is a net contraction of volume. A slide rule that claims to give the amount of water needed for proof reduction was patented by Francis Charles Farmar in 1901[4] (Fig. 2). Farmar[5] boldly declares that it "represents the first really beneficial change made in the slide-rule for 100 years."

Farmar's reduction operation is an example of a heuristic function of two variables: the volume of water z needed to change the proof of one gallon of spirit by some proof amount y to a final proof value of x, i.e., $z(x, y)$, can be calculated from tabulated or computer-fit data. To implement this function on a slide rule, functions f, g, and h must exist that render the composition $h(z)$ additively separable:

$$h(z(x, y)) = f(x) + g(y). \qquad (2)$$

As recently shown,[6] such a separation is not always possible using exact mathematics, but using numerical techniques one may fit arbitrary functions to additively separable approximations, and determine the functions f, g, and h for the approximations in the process.

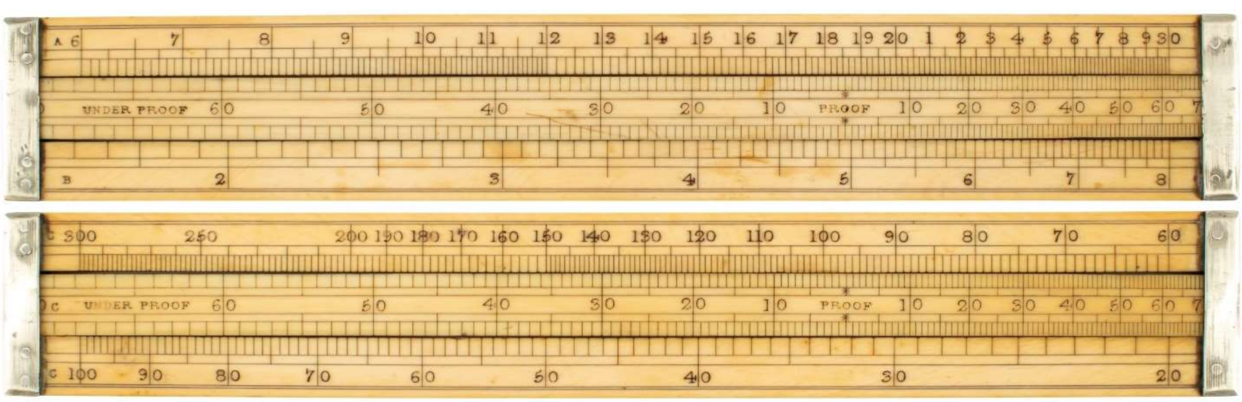

FIGURE 1. A comparative spirit slide rule by Buss, made of ivory with brass braces, nine inches long. Top: the comparative function compares UK Imperial proof on the slider and volume (inverse log scale) on the frame. Bottom: cost per gallon in shillings S at measured proof P obeys $P_1/S_1 = P_2/S_2$. Note that the shillings scale has 12 subdivisions: 12 pence per shilling.

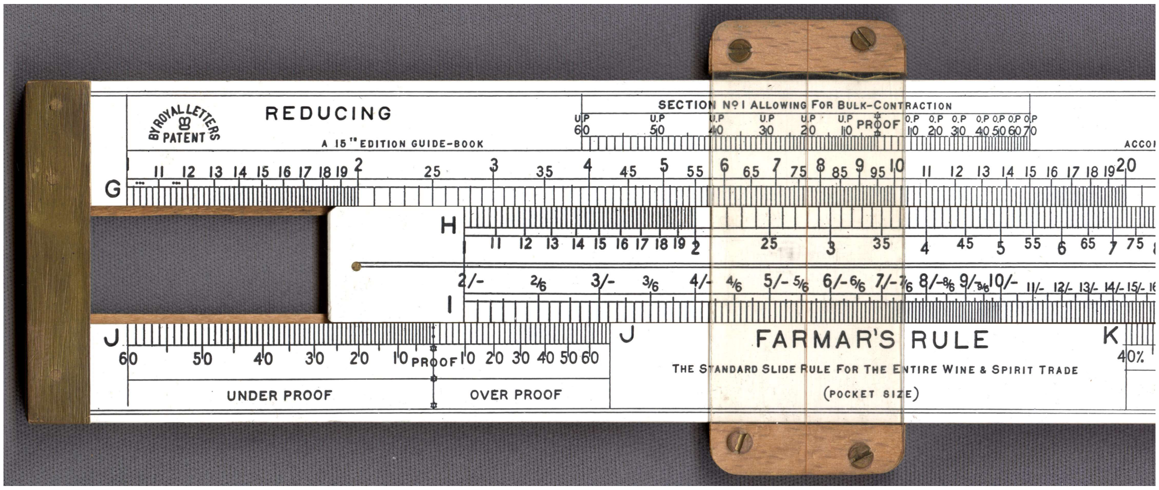

FIGURE 2. A segment of Farmar's slide rule where the reduction function is implemented. Set the difference in proof on line H against the final proof in "Section No. 1." Find the gallons of spirit on line G (between H and Section No. 1), and read the amount of water to add on line H across from the spirit volume. For example, to reduce 50 gallons at 8% over proof to 20% under proof, add 18.2 gallons water: align 28 on H against "20 u.p." on Section No. 1, then find 50 on G (5 with an implicit decimal shift), and find 18.2 across from it on H. This 15th edition "pocket size" slide rule is made of celluloid on wood with brass braces and a plastic cursor, and measures 12¼ inches in length, ca. 1920.

This paper will explore the applications of slide rules to the problem of alcohol reduction or dilution. First, Farmar's reduction calculation will be considered as an example of a heuristic function. Second, after describing the historical context, some antecedents from ca. 1790-1850 will be described. Several hydrometer slide rules from that time contain information sufficient to calculate the reduction formulae. Third, combining the types of data found on historical slide rules, it will be shown that a new reduction slide rule could be made that has no heuristic functions of two variables, and therefore requires no analysis of the type required by Eq. (2).

In detail, formulae like Eq. (1) are strictly true at a prescribed temperature. Similarly, reduction as formulated by Farmar is exact only if water and spirit are at prescribed temperatures. When applied at the wrong temperature some error is incurred beyond the approximations involved in applying Eq. (2) and errors introduced during manufacture and in use. In what follows, temperature will be largely ignored. Calculations to follow assume that the fluid temperatures equal the hydrometer calibration temperature: 55°F before 1817 in the UK and 51°F after, or 60°F in the US after 1851.[7]

Farmar's reduction calculation

The variables chosen by Farmar (Fig. 2) are proof after reduction x, the change in proof y, and the volume of water to be added per volume initial spirit z. The function $z(x, y)$ can be implemented on a slide rule via Eq. (2) if there exist continuous functions $u(x)$ and $v(y)$ such that

$$\left.\frac{\partial y}{\partial x}\right|_z = e^{u(x)+v(y)}. \qquad (3)$$

Does the reduction function obey this equation? If so, the premise of Farmar's reduction calculation is sound, and if not, the reduction calculation on a slide rule will be erroneous regardless of the craftsmanship of the slide rule and its layout.

This question may be addressed by fitting a model of water-alcohol mixtures to a suitable template for u and v, and testing the fit against the model using Eq. (3). To find a template, consider first the behavior of ideal solutions. If alcohol and water mixed ideally, then $z = y/x$, and $\partial y/\partial x|_z = y/x$, so $u = -\ln(x)$ and $v = \ln(y)$. Alcohol and water do not mix ideally, but the ideal character dominates the mixing behavior. Functions u and v are nearly logarithmic, so a good template is

$$u(x) = -\ln(x) + \sum_m c_m p_m(x)$$
$$v(y) = +\ln(y) + \sum_n d_n q_n(y),$$

where p and q are b-splines and the coefficients c and d are to be determined by a fitting procedure.[8]

Choosing the diagonal $x + y = 100$ to be a fiducial line, and evaluating the model with data from the *Organisation Internationale de M´etrologie L´egale* (OIML),[9] contours of the best fit z function are displayed in Fig. 3. In the case of the Dicas hydrometer correction it was found that the underlying $z(x, y)$ model was incompatible with the form of Eq. (3) in part of the domain.[10] No such discrepancy is found here: the underlying physical behavior is well-approximated by Eq. (3) and spirit reduction is therefore amenable to accurate calculation with a slide rule.

Note that in Fig. 3 the contours of constant z, labeled from 0 (horizontal axis) to ∞ (vertical axis) are nearly rays emanating from the origin. Rays would be the expected behavior if mixing were ideal, $z = y/x$. The departure from ideal mixing behavior can be detected in the slight curvature of isopleths as they approach the $x + y = 175$ diagonal limit of the figure. Although slight, accurately accounting for this nonideality is the point of the reduction calculation.

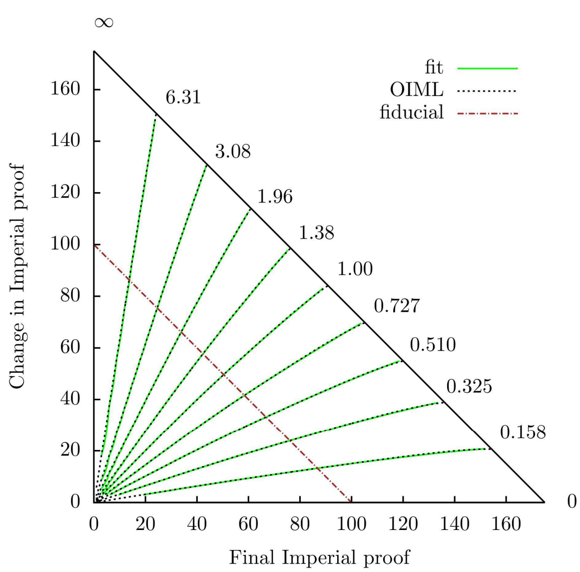

FIGURE 3. Contours of the volume of water needed per volume of initial spirit to reduce proof, at 51°F. Dashed black curves are obtained by calculation using the OIML data set. Solid green curves are a best fit additively separable function. The diagonal brown (dot-dash) line is the fiducial line used to assign function values to isopleths.

Graphically, the black dashed contours in Fig. 3 calculated from the OIML data set, and the green solid contours calculated with Eq. (3) using best fit u and v, are indistinguishable. They are not exactly identical, however. Figure 4a displays error contours of this best fit. Near the y axis the errors appear to be large, but as $z \to \infty$ on that axis, the relative errors are quite small. The error is zero on the fiducial curve, so selection of a different fiducial would influence the distribution of errors. On a 45° ray $z \approx 1$, so it is easy to see that away from $x = 0$ the maximum magnitude of the relative error is in the neighborhood of 0.1% along that ray. The functions f, g, and h associated with this best fit are close to being logarithmic because, as noted, $z \approx y/x$. The deviation from logarithmic behavior is responsible for modeling the curvature of isopleths in Fig. 3.

Farmar's slide rule goes beyond calculating the ratio of water to spirit needed: it places that result on a logarithmic scale to facilitate multiplication. If z gallons of water are indicated for 1 gallon of initial spirit, then z is found adjacent to 1, and so too is $2z$ found adjacent to 2, etc. This convenience comes at a cost: for multiplication to be automatic, the scale h must be logarithmic. Further, his line of proof change $g(y)$ coincides with his reduction water line $h(z)$, so both $g(y) = \ln(y)$ and $h(z) = \ln(z)$. This places a constraint on Farmar's fit that was not contemplated in calculating the best fit result. Rather than finding f, g, h to optimize the representation of $z(x, y)$ on a slide rule, Farmar made the *ad hoc* choice $h = g = \ln$ and could only adjust the line of final proofs f.

Figure 4b displays the errors associated with a best fit of an OIML calculation to Farmar's logarithmic constraint model. Again, using the 45° ray where $z \approx 1$ as a guide, one sees that relative errors are in the neighborhood of 0.1% when $y \leq 60$ but closer to 1% as y approaches 80. It can be seen that Farmar's approximation introduced greater error that was absolutely necessary, but Farmar's approximation unquestionably makes the slide rule easier to use. The actual error associated with Farmar's slide rule is naturally larger than this theoretical best fit calculation, but for proofs above 50°IP they are less than an order of magnitude greater than the theoretical Fig. 4b result. Farmar's slide rule is less accurate for lower proofs.

In addition to the "Section No. 1" calculation shown in Fig. 2, from the 15th edition of Farmar's rule there is a similar calculation supported on the opposite side of the slide rule denominated in ABV. Find the final strength in ABV on line AA,

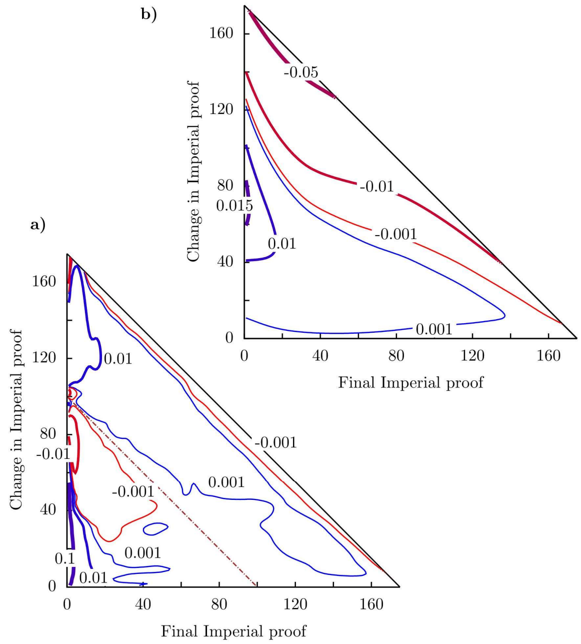

FIGURE 4. (a) Contours of the error $z_{fit} - z_{OIML}$ comparing the best fit and the OIML calculations of water needed per volume initial spirit to reduce proof. Error is zero on the brown (dot-dash) fiducial line. (b) Contours of error of a fit that assumes $h = g = \ln$ but where f is free to vary.

and move the slider to align with it the change in ABV on line B. Next, locate the initial volume on line A, and find across from it on line B the volume of water to add. This operation is completely analogous to the Imperial proof version of the calculation.

Earlier editions of Farmar's rule have an AA line which serves a different purpose. From the 1902 edition of Farmar's guide[11] this line is related to the evaluation of Eq. (1). Locate the initial proof P_1 on line AA and set it against the proof after reduction P_2 on line BB. Then, against the initial volume V_1 on line B find the volume after reduction V_2 on line A.

Later editions of Farmar's rule have a line marked "Section No. 2 Not Allowing For Bulk Contraction." Its use is analogous to Section No. 1. Final proof is located on the Section No. 2 line, and the change of proof is placed across from it on slider line H. Find the volume of spirit on frame line G, then read the incorrect volume of water on line H. This calculated volume equals $V_2 - V_1$ from Eq. (1). In fact, rearranging Eq. (1) one has

$$\frac{P_1 - P_2}{P_2} = \frac{V_2 - V_1}{V_1}$$

which explains why lines G, H, and Section No. 2 are all logarithmic.

Other calculations supported by Farmar's spirit rule, and other slide rules by Farmar, are described by Barnes.[12,13]

A digression on excise reform

To set the stage for a discussion of antecedents to Farmar's slide rule it is helpful to understand how alcohol strength was measured before and after 1817 – a pivotal date in a period of alcohol excise tax turmoil that culminated in a sweeping reform in 1823. The instruments used changed little in this period, but the way alcohol was measured changed significantly. Details of this change, recorded on slide rules, can be used to compute reductions.

Invented ca. 1725,[14,15] Clarke's hydrometer was the *de facto* standard for measuring the strength of spirits in the UK until 1787 when its status was made official.[16] However, its elevation in status was at odds with its reputation in the excise and trade. It was known to be inaccurate and internally inconsistent, and to illogically treat spirits of different origin differently.[17,18,19] Its reputation was so dissonant with its status that, when its official status was renewed in 1789,[20] MP Richard Brinsley Sheridan declared that Parliament had "sanctioned error, and legalized falsehood and oppression."[21] The "whereas" clause of the 1787 act called for new experiments on water-alcohol mixtures, and in the early 1790s these were conducted by Royal Society fellows Charles Blagden and George Gilpin.[22, 23, 24] But, without accounting for the new experiments, Parliament made Clarke's status "perpetual" in 1801.[25]

The union of Great Britain and Ireland in 1800 prompted a reexamination of hydrometers and the excise.[26] In 1802, a very modest newspaper notice[27] informed the public that the Commissioners of Excise were ready to entertain offers for new instruments for ascertaining the strength of spirituous liquors. Accounts of the resulting 1803 competition emphasize ease of use and comparative accuracy as the basis for selecting the new UK standard hydrometer, and against those criteria the selection of Sikes' hydrometer was a peculiar choice. Mechanically, the 9-weight Sikes hydrometer appeared to be a compromise between the simple 4-weight "centesimal" Dring & Fage entry, and the more complex 36-weight Dicas entry, to name but two of Sikes' competitors. Yet, the Dicas and Dring & Fage designs offered speedy computation with customized slide rules whereas the Sikes entry relied on far more cumbersome tables. Sikes' hydrometer was made the UK official hydrometer in 1817.[28] (Hydrometers other than Sikes' were permitted in 1907,[29] and the Sikes system was abandoned altogether in 1979 after the UK joined the European Union.[30])

While Sikes' design was a mere compromise mechanically, it was quite innovative in how it was calibrated. The standard of measurement used by Clarke, Dicas, and others was the fraction over or under proof in a water-oriented sense: if a spirit were "1 to 6 over proof" then to 6 gallons of spirit, the addition of 1 gallon of water would make the spirit be at proof. If the spirit were "1 in 5 under proof" then from 5 gallons of spirit, remove 1 gallon of water to make proof spirit. (Calculating reductions to proof was trivial with this system, but not to any other strength.) Sikes' system used more modern decimal numbering and used an entirely different scale based on the volume of alcohol contained in the mixture: one gallon at 10% over proof has the same alcohol content as 1.1 gallons at proof. This came to be seen as an advantage to the trade, but it was mainly an advantage for the excise. (Clues that the Treasury would value a new alcohol volume-based system of measurement can be found before the hydrometer competition, e.g., "it was thought, from the first, that the best method of adjusting the duty would be by the absolute quantity of alcohol in any mixture."[31] Bartholomew Sikes, a former Secretary to the Board of Excise, likely appreciated these clues.[32])

In Ireland, where hydrometers had rarely been used outside of ports, distillers were taxed using indirect proxies of the alcohol produced.[33,34] From 1779[35] Irish distillers were taxed for the volume of wash distilled, and from 1785 they were also taxed for the malt they used. The malt tax was discontinued upon Union, but the wash tax continued. Still capacity was used as a proxy for wash consumption, and arbitrary assumptions about the number of wash distillations conducted per day made excise tax only vaguely related to the amount of spirit produced. In Scotland, from 1784 until 1823 fees were assessed based on the gallon capacity of stills, regardless of how often they were used.[36] In 1823[37] the proxies were abandoned, and distillers were taxed according to the number of gallons produced on a hydrometer proof basis. This new rational tax system was made possible by Sikes' new alcohol-volume-based system of measurement.

The 1816 act that adopted Sikes' hydrometer as the new standard redefined proof spirit. It was now to be that mixture of water and alcohol whose specific gravity is 12/13 at 51°F. It is noteworthy that this blend, and the UK's prior standard, have alcohol mass fractions of nearly 50%.[38,39] Nettleton[40] thinks that "it would appear to be the outcome of an attempt to produce a mixture of pure alcohol and water, in which there would be equal weight of the two components." The new definition is also different from "specific gravity 0.920 at 60°F," which is what the Board of Excise told the contestants of 1803.[41,42]

The 1816 Sikes act also calls for the creation of slide rules for "ready calculation of the quantity of spirits of certain strengths ... contained in or which can be made from any quantity of spirits of any other strength," evidently in anticipation of the 1823 excise law. The comparative slide rule (Fig. 1) performs the calculation required by this act. A subsequent 1818 act[43] repealed the 1816 act, including its call for slide rules. It reaffirmed the adoption of Sikes' hydrometer, but called for the use of a book of tables.

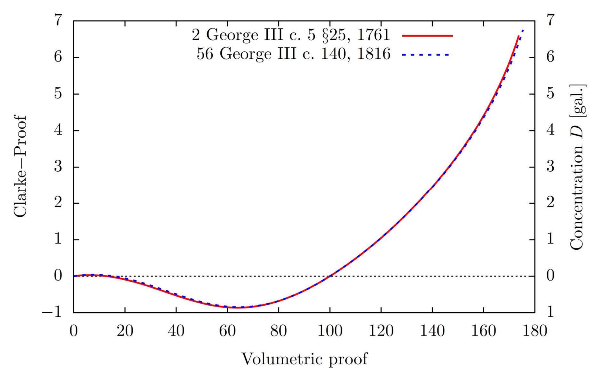

FIGURE 5. A calculation of the relation between the Clarke/Dicas type measure of strength and the associated volumetric proof using two definitions of proof spirit.

Computing reduction before Farmar

"Contraction" aka "diminution"

Around 1800 the word "concentration" was, in some circles, used to describe the contraction (a better word) in volume that takes place when water is added to spirit.[44] The word "diminution" has also been used to describe that volume deficit. Concentration was reported as the size of the effect in gallons when 100 gallons of spirit are converted to proof. Concentration, the Clarke/Dicas strength, and proof are related, and together can be used to compute a reduction.

Let C be a measure of alcoholic strength in the Clarke/Dicas sense, e.g., if $C = 110$, or 10% over proof, then to 100 gallons of spirit add 10 gallons of water to make the mixture be at proof. Let D be the diminution or concentration. If 100 gallons of strength C spirit were reduced to proof, the final volume would be $C - D$. Finally, let P be a measure of alcoholic strength in the modern proof sense: a quantity proportional to alcohol by volume, with 100°P being the standard of proof spirit. Because the volume of alcohol is conserved in a reduction, 100 gallons of spirit at proof P gives P gallons at 100°P: the volume at proof is $P = C - D$. Over proof, $P \leq C$.

If $C = 90$, or 10% below proof, then to 100 gallons of spirit subtract $10 = 100 - C$ gallons of water to make the mixture be at proof. From Eq. (1), 100 gallons at proof P makes P gallons at 100°P. Instead of subtracting water, the same $(100 - C)$ gallons of water could be added to the P gallons of proof spirit to make 100 gallons of under proof spirit. The spirit volume, initially P, plus the water volume $(100 - C)$, less the contraction D, equals the under proof volume 100: $P = C + D$. Underproof, $P \geq C$ except for very weak spirits where $D < 0$.

Figure 5 shows the difference between the Clarke/Dicas measure of strength and a volumetric proof measure, with both measures sharing the same definition of proof spirit. Whether the definition is the one originally applicable to Clarke (solid red), or the one instituted with the adoption of Sikes (blue dashed) makes little difference in the numbers. The absolute value of this difference is the concentration. Note that for under proof spirits $P > C$ except for proofs less than about 10°P. When water and pure alcohol are mixed in any proportion the mixture occupies less volume than the sum of the ingredients. However, when water and proof spirit are mixed the contraction is not observed for every proportion. This is because at very low strengths the partial molar excess volume of water is positive.[45] The molar excess volume of mixing is everywhere negative, but it is not strictly convex.

Knowing the Clarke/Dicas measure and the proof strength, one can contemplate how much water is needed to reduce volume V_1 of spirit at proof P_1 to volume $V_2 = V_1P_1/P_2$ at proof P_2. The calculation takes advantage of a fictitious proof spirit intermediate fluid. The two scales may be calibrated to different ideas of proof. Let P_C denote the proof-scale measurement of the Clarke/Dicas scale's proof definition. With these definitions, 100 gallons at proof P_1 plus $C_1 - 100$ gallons of water gives $100P_1/P_C$ gallons of spirit at proof P_C. Therefore, V_1 gallons at proof P_1 plus $V_1(C_1 - 100)/100$ gallons of water will give V_1P_1/P_C gallons of spirit at proof P_C. Likewise, V_2 gallons at proof P_2 plus $V_2(C_2 - 100)/100$ gallons of water will give V_2P_2/P_C gallons of spirit at proof P_C Setting these mixtures at P_C to be equal, one deduces that adding

$$V_W = \frac{V_1}{100}(C_1 - 100) - \frac{V_2}{100}(C_2 - 100) \tag{4}$$

gallons of water to condition 1 gives condition 2. Note that the calculation requires Clarke/Dicas and volumetric-proof type measures (to relate V_1 and V_2), but it is independent of the value P_C.

In 1781 Richard Clarke's business passed on to his son-in-law John Dring, and by 1789 to the firm of Dring & Fage. Figure 6 displays a slide rule by Dring & Fage that accompanied their "centesimal" Clarke hydrometer. References to this hydrometer date from 1799.[46,47] Clarke hydrometers other than the centesimal model used attached "air weights"[48,49] (aka "weather weights"[50,51]) to correct hydrometer readings for the effect of temperature. The centesimal model used the slide rule to perform the correction.

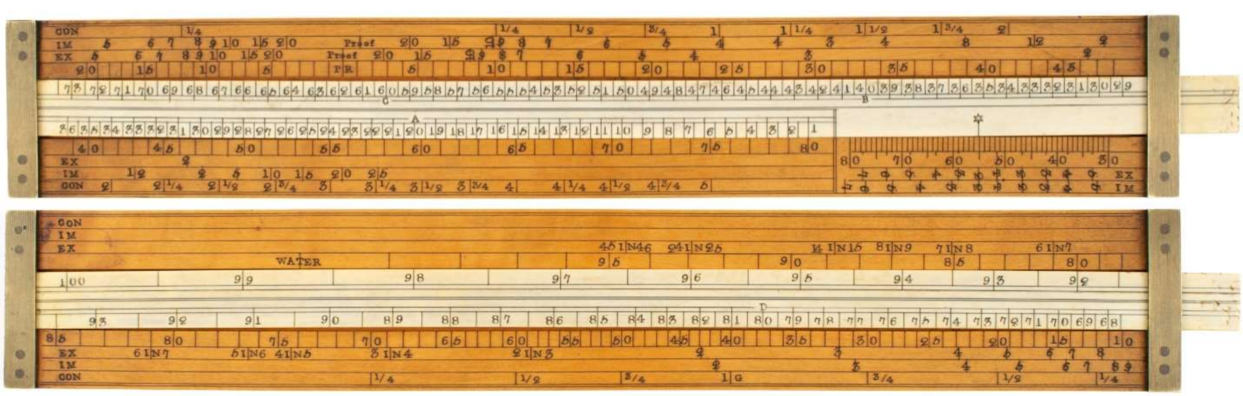

FIGURE 6. Dring & Fage centesimal hydrometer slide rule. Boxwood with ivory slider and brass braces, ten inches long.

Before the centesimal model, Clarke hydrometers used a number of screw-on weights to enable the hydrometer to operate across a wide range of spirit strengths. The stem of the hydrometer gave a measure of departure from the strength indicated on the weights, but being say n tick marks stronger than "1 to 9 over proof" couldn't be related to m tick marks weaker than "1 to 8 over proof:" the old hydrometer did not relate measurements with different weights to a single continuous line of spirit strengths.[52] The new centesimal hydrometer put all measurements onto a single line of instrument readings from 0 to 100. These instrument readings appear on the slider in Fig. 6. On the frame are a new measure of volumetric proof, lines of Clarke strength, and a line of concentration "CON."

A peculiarity of Clarke's hydrometer is that domestic and exported spirits and imported spirits were measured with different instruments. The exact origin of this practice is uncertain, but Owens[53] implies that it was an attempt to deal with the problem of obscuration: the density-increasing effect of sugar, commonly found in imported spirits, competes with the density-lowering effect of alcohol. Excise Commissioners first identified this as a serious problem for imported spirits in 1760,[54] although specialized measurements for arrack had been previously devised by Clarke[55] and by Martin.[56] The centesimal slide rule accordingly has two Clarke scales, one marked "EX" for domestic and export, the other marked "IM" for import. On the over proof side of these scales, a number like "5" stands for "1 to 5 over proof." Integers on the under proof side have corresponding meaning, e.g., "5" stands for "1 in 5 under proof." For very dilute spirits the labels are more complete. The most dilute indication is "45 in 46."

The centesimal slide rule also has marks beneath the temperature scale indicating which air weight should be used for the export or the import measurement. On

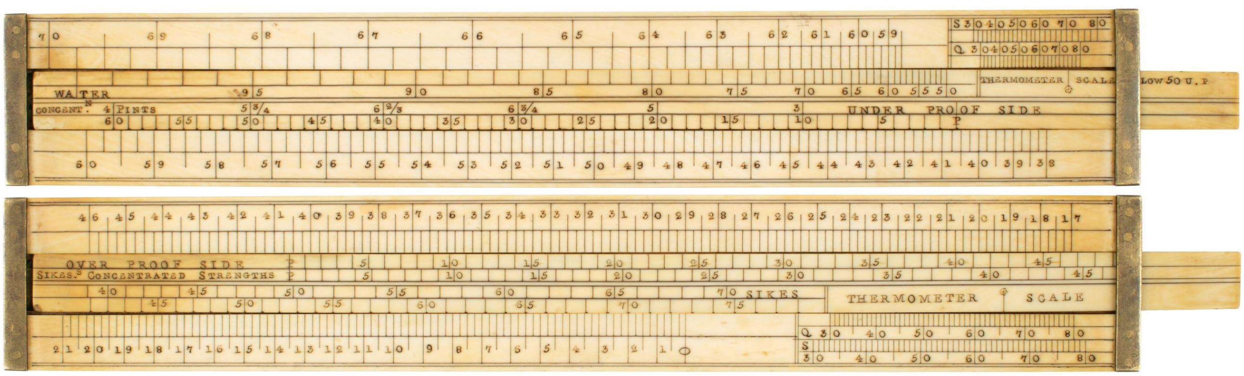

FIGURE 7. An ivory slide rule by John & George Quin with brass braces, 9 7/16 inches long.

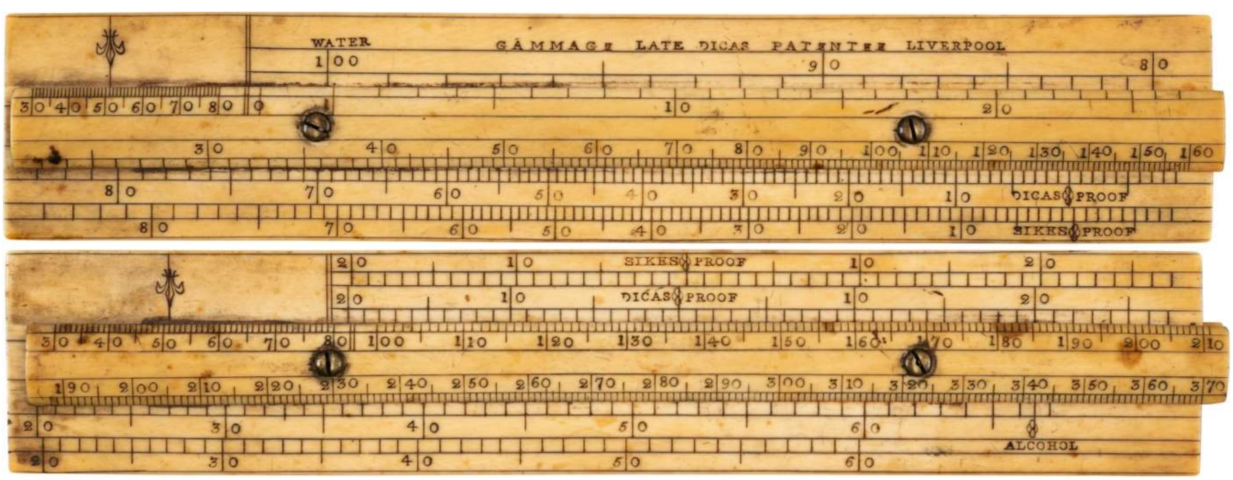

FIGURE 8. A slide rule made ca. 1833—1851 by Benjamin Gammage, son-in-law and successor to John Dicas, for the temperature correction of Dicas style hydrometers. The slider gives the instrument reading from 0 to 370. The frame shows alcohol strength using both Dicas and Sikes scales. The slide rule is ivory and measures 7 7/16 inches long.

the export hydrometer these are[57] (5) “very cold” ≤ 32°F; (4) “colder” 32–37; (3) “cold” 37–41; (2) “coldish” 41–46; (T) “temperate” 46–51.5; (2) “warmish” 51.5–55; (3) “warm” 55–60; (4) “warmer” 60–64; (5) “hot” 64–69; (6) “hotter” 69–74; and (7) “very hot” 74–80. The import hydrometer uses the same numbering and naming system, but the temperatures are shifted. (It is not clear how these alternative temperature lines would be used. Perhaps they were for the benefit of gaugers who could judge that it was (say) “warmish” but lacked a thermometer.)

Blagden[58] believed the standard of temperature in the trade to be 55°F. This Dring & Fage slide rule is consistent with that belief: when this slide rule is indexed to 55°F a vertical decoration separating the proof scales from the temperature scales aligns with a vertical decoration on the slider.

This Dring & Fage slide rule has both P and C lines. The D line is redundant since $D = |C - P|$. Since there is a single D line and a single P line, these equations cannot work equally for the two C lines – the export C line is most consistent. The P and export C lines contain all the information needed to interpret reduction Eq. (4).

Figure 7 displays a slide rule by John and George Quin for use with a hydrometer. The frame has an instrument scale numbered from 0 to 70, and two temperature scales: “Q” for Quin and “S” for Sikes. (Different systems of measurement by the same instrument would use the same temperature scale, so the existence of two scales is perplexing. Also puzzling, the edge of the slide rule is engraved “J & G Quin Patentee,” yet they are not recorded as having been awarded patents of invention.[59]) The slider has two measures of alcoholic strength. On the over proof side, the scale marked “Sikes concentrated strength” is evidently the Sikes volumetric proof. The other over proof scale, which is unmarked, appears to be a decimal Clarke/Dicas type scale (vs. one denominated in fractions like “1 to 9”) based on the same proof spirit strength as Sikes. The under proof side has the Clarke/Dicas type scale and a line of concentrations. If these interpretations are correct, C and P lines on the over proof side, or C and D lines on the under proof side, provide the information needed to evaluate the reduction formula Eq. (4).

A different slide rule by John and George Quin appearing in the Tom Wyman collection[60] also computes both proof and concentration. In 1814 George Quin[61] described yet another slide rule (for a “new patent hydrometer” – another reference to a phantom patent) which contains a Clarke/Dicas scale, a scale of diminution, and a specific gravity scale.

Mary Dicas, who succeeded her father John in 1797, represented the family business in the 1803 Excise review of hydrometers. She produced hydrometers with her husband George Arstall from 1807 to 1811, then the business passed to her sister Ann in 1818. Ann was succeeded by her husband Benjamin Gammage from 1833 to 1851.[62] Gammage’s Dicas hydrometer slide rule (Fig. 8) bears a Dicas scale and a Sikes scale, and it is clear from the location of the proof indicators that the notion of proof spirit is different for these scales. Nonetheless, using the Sikes scale as the volumetric P measure, the Gammage slide rule also contains all the information needed to compute a reduction using Eq. (4). An 1792 encyclopedia article describes a Dicas slide rule which included a line of concentrations[63] (and which predates Blagden and Gilpin’s main publications).

Specific gravity

Another way to supply the information necessary to perform a reduction is to augment a measure of spirit strength with the specific gravity ρ. From the conservation of total mass,

$$V_W = V_2\rho_2 - V_1\rho_1 \qquad (5)$$

is the volume of water needed to dilute state 1 to state 2, where the spirit volumes are related by conservation of alcohol, Eq. (1).[64]

The specific gravity, the Clarke/Dicas strength, and proof are related: any two determine the third. Again, let C be the Clarke/Dicas measure of alcohol strength, and let ρ be the specific gravity. Then V_1 gallons at spirit measure C_1 and specific gravity ρ_1, plus $V_1(C_1 - 100)/100$ gallons of water, will give a proof spirit with weight proportional to $V_1(\rho_1 - 1 + C_1/100)$. Likewise, V_2 gallons at C_2 with specific gravity ρ_2, plus $V_2(C_2 - 100)/100$ gallons of water, will give a proof spirit with weight proportional to $V_2(\rho_2 - 1 + C_2/100)$. Equating the two proof spirit weights, the volume of reduced spirit is found to be

$$V_2 = V_1 \frac{100(\rho_1 - 1) + C_1}{100(\rho_2 - 1) + C_2} \qquad (6)$$

and the amount of water necessary is again given by Eq. (4).

To determine proof P, let ρ_C be the specific gravity of proof spirit by the Clarke/Dicas measure, and recall that P_C is its volumetric proof measure. A mixture of 100 gallons at Clarke/Dicas measure C plus $(C - 100)$ gallons of water has weight proportional to

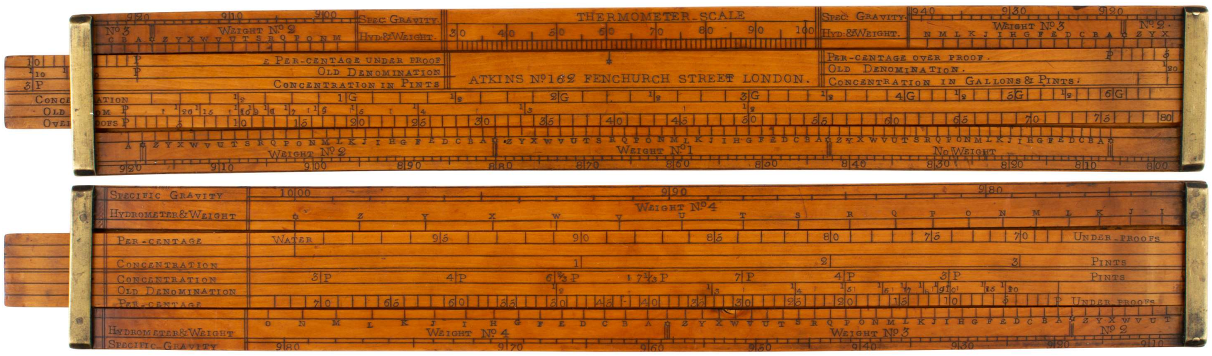

FIGURE 9. Atkins' slide rule based on Matthew Quin's design. Made of boxwood with brass braces, ten inches long.

$100(\rho - 1) + C$, and has a volume $[100(\rho - 1) + C]/\rho_C$. Therefore, the volumetric proof before reduction would be given by

$$P = \frac{P_C}{\rho_C}\frac{100(\rho - 1) + C}{100}. \quad (7)$$

This relation, with Eq. (1), is another way to derive Eq. (6). Using Eq. (7) to eliminate specific gravity from Eq. (5) yields Eq. (4) upon simplification.

A slide rule by Atkins (Fig. 9) displays "old denomination" alcoholic strength in fractions, decimal representations of the same, and "concentration" on the slider. The frame shows "hydrometer weight" in increments marked by a star symbol followed by letters A to Z, broken into five lines depending on which of four possible weights is attached to the hydrometer, or none. The frame also has a 30–100°F temperature scale and a line of specific gravities. Since specific gravity is on the frame it is not subject to temperature correction. The alphabetic instrument scale suggests that this slide rule is one of Atkins' productions of Matthew Quin's slide rule for his universal hydrometer.[65] (Robert Atkins and Matthew Quin were business partners under the name Atkins & Quin ca. 1799.[66]) Quin's hydrometer had alphabetical indications on the stem for use with spirits, and numeric indications for use with worts, so this slide rule is intended only for use with spirits. Placing the temperature indicator on the frame against 60°F aligns "water" with specific gravity 1.000, suggesting that Atkins used 60°F as a calibration temperature. Indexed to 60°F, the specific gravity lines, the concentration lines, and the alcoholic strength (Clarke/Dicas) can be interconverted. This would permit the estimation of reduction water volume using P and ρ using Eqs. (1) and (5); using C and P with Eqs. (1) and (4); or using C and ρ with Eqs. (4) and (6).

Fletcher[67] shows a plate of an Atkins hydrometer and its slide rule. Its layout is a bit different from Fig. 9, and it includes both Clarke and Dicas scales in addition to Atkins' proof scale, and concentration. It too has alphabetic instrument readings, a temperature scale, and a specific gravity line on the frame. Atkins and Co.[68] describe yet another slide rule which contains the same information as the Dring & Fage one (Fig. 6).

From these examples it is clear that the new experiments of Blagden and Gilpin had an impact on instrument makers in the 1790–1817 time frame. The idea of a volumetric proof scale was the most significant and lasting change from this time. The associated idea of concentration clearly had appeal, as did the fundamental concept of specific gravity. In their 1803 book, Atkins & Co.[69] show how to solve dilution-related problems using knowledge of a Clarke/Dicas measure and the associated specific gravity. They also provide meaningful and correct examples on the use of concentration. Yet, around this same time (1806) Jonas[70] claims to show the quantity of spirit obtained by dilution while neglecting the concentration. The limitations of the old Clarke/Dicas system were clearly not universally appreciated at that time. And, despite their appreciation by Atkins, neither he nor his peers presented this information in a particularly useful format. The lines of numbers on these slide rules are essentially auxiliary data that, like tabulated data, have to be manipulated with paper and pencil to be profitably applied. It could have been done differently.

An exact reduction slide rule

Rearranging Eq. (4) somewhat using Eq. (1) to eliminate V_2, one finds that to reduce V_1 gallons at proof P_1 to proof P_2 it is necessary to add

$$V_W = \frac{V_1 P_1}{100}\left(\frac{C_1 - 100}{P_1} - \frac{C_2 - 100}{P_2}\right) \quad (8a)$$

gallons of water. Beginning with the specific gravity formulation of Eq. (5), the equivalent expression,

$$V_W = \frac{V_1 P_1}{100}\left(\frac{100\rho_2}{P_2} - \frac{100\rho_1}{P_1}\right) \qquad \text{(8b)}$$

is obtained.

When P is measured with the US 0–200 proof scale and V is measured in gallons, $V_1 P_1/100$ is the amount of alcohol contained in the spirit in units of "proof gallons," which is how alcohol abundance is measured for regulation by the US Alcohol and Tobacco Tax and Trade Bureau (TTB).

The quantities $(C - 100)/P$ (equivalently $-100\rho/P$) can be put on a single line of numbers. In contrast to bivariate functions $z(x, y)$, univariant functions $f(x)$ like this one act as simple look-up tables. There is no obstacle to implementing them on a slide rule.

Figure 10 illustrates how the term in parenthesis in Eq. (8) could be evaluated on a slide rule. The top rule is labeled by the US proof, with P marks spaced to be linear in $(C - 100)/P$. The bottom rule is scored in the same $(C - 100)/P$ linear increments. Align zero on the bottom scale with the final strength on the top scale (here 80°P), then read a number on the bottom scale across from the initial proof strength (here 0.554 across from 140°P). To complete the calculation, use Gunter's rules to multiply this result by the alcohol quantity in proof gallons. Twenty gallons at 140°P is 28 proof gallons. To reduce 20 gallons at 140°P initial strength to a final strength of 80°P one requires $28 \times 0.554 \approx 15.5$ gallons of water at 60°F.

The calculation of reduction using reliable data has been possible since ca. 1794, but the construction of a specialized slide rule apparently did not occur until Farmar in 1901. Why were reduction slide rules not made earlier? It is likely that there was little demand. In the 19th century most spirits were distributed in casks, then sold and consumed in pubs where reduction was done informally.[71] Alcoholic strength was not regulated in the UK until 1879 when only a minimum proof was imposed.[72] When sold above that minimum, no declaration of strength was required, so there was no particular commercial need for accurate reduction. In the US a declaration of alcoholic strength on alcoholic beverage labels was not required until 1936,[73] and in the UK it was not required until 1946.[74] For those desirous of accuracy despite there being no legal need, in 1890 Sheridan[75] published a user-friendly reduction guide. In the early 20th century, later editions of Sheridan's Reducing Table were widely circulated free to all subscribers to the *Wine & Spirit Trade Record,* a leading trade publication.

Conclusions

Viewed as an empirical function of two variables, spirit reduction can be implemented on a slide rule with modest (ca. 0.1%) errors over much of the parameter range. The most accurate implementation uses non-logarithmic scales for the inputs (final proof, change in proof) and the output (volume water per initial volume spirit), and requires an auxiliary computation (multiplication by initial spirit volume). Farmar's slide rule uses logarithmic scales for the change in proof and for the result, which makes the final multiplication automatic. Farmar's design thereby greatly enhances ease of use, but at the cost of accuracy.

From ca. 1790 to ca. 1817 instrument makers in the UK began to incorporate data on hydrometer slide rules that allow alcoholic strength to be expressed in terms of the volume fraction of alcohol – modern proof scales. With proof scales alone one cannot compute the amount of water needed for a reduction.

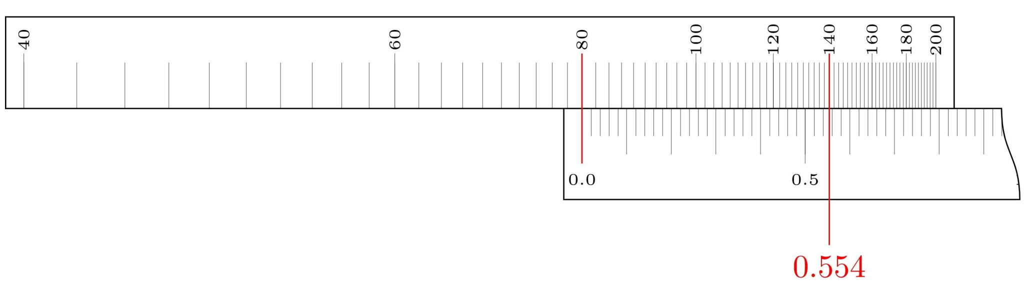

FIGURE 10. An "exact" reduction slide rule can be made using a line of numbers derived from the ratio of a Clarke/Dicas measure of alcoholic strength to a proof measure based on alcohol volume fraction. To reduce one proof gallon at 140°P (US) to 80°P, add 0.554 gallons of water. An animated version of the complete "exact" slide rule can be found on JOS Plus, on the Oughtred Society website.

However, several slide rules made during this transition period, and after, incorporated both new and old measurement scales, and in combination those scales may be used to compute the amount of water needed for a reduction. Slide rules with lines of alcohol strength and lines of specific gravity also supply the necessary information. An "exact" reduction slide rule can be made from a line of numbers combining a Clarke/Dicas scale and a volumetric proof scale.

The comparative slide rule, which bears the proof scale only, cannot be used to calculate the volume of water needed for a reduction. That was not its purpose. Nevertheless, it can be used to accurately perform a reduction if used properly, as could any logarithmic slide rule. For example, to make V_2 gallons of spirit at proof P_2 beginning with a proof P_1 spirit: place volume $V_1 = V_2 P_2 / P_1$ of the P_1 spirit in a vessel of volume V_2 then fill to the V_2 mark with water.

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Notes

1. "*An Act Imposing Taxes on Distilled Spirits and Tobacco, and for Other Purposes*," July 20, 1868.
2. P. Vander Meulen, "*Sikes's hydrometer and related slide rules,*" Journal of the Oughtred Society, 10:1, 2001, pp. 30–34.
3. T. Martin, "*Hydrometry and slide rules in brewing and distilling,*" Proceedings of the 11th International Meeting of Slide Rule Collectors, October 2005, pp. 51-67. Reprinted in Journal of the Brewery History Society, 124, 2007, pp. 15–38.
4. Patents GB 1901 no. 9309 and US no. 788638 1905.
5. F. C. Farmar, *Guide to Farmar's Rule. The Recognized Standard Slide-Rule for the Entire Wholesale & Retail Wine and Spirits Trade*, 14th ed., 1919.
6. G. Miller, "*Empirical functions of two variables, and Dicas' hydrometer correction,*" Journal of the Oughtred Society, 30:1, 2021.
7. Farmar's UK patent (GB 1901 no. 9309) mentions 62°F as a standard temperature when describing a line to calculate density in pounds per gallon from proof, but there is no indication in his patents or his booklet (Farmar, *Guide*) that reduction should be carried out at a particular temperature.
8. Miller, "*Empirical functions.*"
9. Organization Internationale de Métrologie Légale, Recommendation 22, International Alcoholometric Tables, Paris, 1975.
10. Miller, "*Empirical functions.*"
11. Farmar, *Guide*, 1902.
12. C. Barnes, "*Farmar's rules,*" Slide Rule Gazette, 15, 2014, pp. 79–92.
13. C. Barnes, "*Farmar's rules – Contd*.," Slide Rule Gazette, Colin Barnes Memorial Issue, 2016, pp. 6–8.
14. J. T. Desaguliers. "*A new kind of hydrometer made by Mr. Clarke,*" Philosophical Transactions of the Royal Society of London, 36, 1729, pp. 277–279.
15. F. G. H. Tate and G. H. Gabb, *Alcoholometry*, London: His Majesty's Stationery Office, 1930.
16. 27 George III c. 31, §17, 1787.
17. W. J. Ashworth, *"Between the trader and the public: British alcohol standards and the proof of good governance,"* Technology and Culture, 42, 2001, pp. 27–50.
18. W. J. Ashworth, *Customs and Excise: Trade, Production, and Consumption in England 1640–1845*, Oxford: Oxford University Press, 2003.

19. Atkins & Co, *An Essay on the Relation Between the Specific Gravities and the Strengths and Values of Spirituous Liquors*, London: Knight & Compton, 1803.
20. 29 George III c. 55, 1789.
21. R. B. Sheridan, *Speeches. Vol. II,* London: Patrick Martin, 1816, p. 258.
22. C. Blagden, *"Report on the best method of proportioning the excise upon spirituous liquors,"* Philosophical Transactions of the Royal Society of London, 80, 1790, pp. 321–345.
23. C. Blagden and G. Gilpin, *"Supplementary report on the best method of proportioning the excise upon spirituous liquors,"* Philosophical Transactions of the Royal Society of London, 82, 1792, pp. 425– 455.
24. G. Gilpin and C. Blagden, *"Tables for reducing the quantities by weight, in any mixture of pure spirit and water, to those by measure; and for determining the proportion, by measure, of each of the two substances in such mixtures,"* Philosophical Transactions of the Royal Society of London, 84, 1794, pp. 275–382.
25. 41 George III c. 97 §8, 1801.
26. 42 George III c. 97, 1802.
27. E.g., London Gazette, 25 August 1802, p. 901.
28. 56 George III c. 140, 1816.
29. 7 Edward VII c. 13, 1907.
30. 1979 no. 241, "Alcoholic Liquors Order 1979."
31. Blagden and Gilpin, *"Supplementary report,"* p. 429.
32. Tate and Gabb, *Alcoholometry,* p. 6.
33. J. S. Rochfort, F. Geale, R. Alexander, and C. Saxton, *The Fifth Report of the Commissioners Appointed to Enquire into the Fees, Gratuities, Perquisites, and Emoluments, which are or have been lately received in certain Public Offices in Ireland; and also to examine into any Abuses which may exist in the same; and into the present Mode of Receiving, Collecting, Issuing, and Accounting for Public Money in Ireland*, House of Commons 124, 1807.
34. E. B. McGuire, *Irish Whiskey. A History of Distilling, the Spirit Trade and Excise Controls in Ireland*, New York: Harper & Row, 1973.
35. 25 George III c. 3, 1785, Irish Parliament.
36. G. Miller, *Whisky Science: A Condensed Distillation*, New York: Springer, 2019.
37. 4 George IV, c. 94, 1823.
38. J. Drinkwater, *"On the preparation of absolute alcohol, and the composition of 'proof-spirit',"* The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 37, pp. 123-129, 1848.
39. J. Scarisbrick, *Spirit Assaying, Revenue Series No. 3,* 2d ed., Burton-on-Trent: J. N. Tresise, 1898.
40. J. A. Nettleton, *The Manufacture of Spirit,* London: Marcus Ward & Co., Ltd., 1893.
41. G. Quin, *A Descriptive Account of the New Patent Hydrometer*, London: W. Clowes, 1814, p. 10.
42. Scarisbrick, *Spirit Assaying*.
43. 58 George III c. 28, 1818.
44. J. Fletcher, *"Description of Atkin's hydrometer for ascertaining the specific gravities of spirituous liquors,"* Journal of Natural Philosophy, Chemistry, and the Arts (Nicholson's Journal), 1802, pp. 276– 281,304.
45. Miller, *Whisky Science,* §9.14.
46. S. Douglas, *"Report from the Committee Upon the Distilleries in Scotland, 12 July 1799,"* Reports from Committees of the House of Commons 1782–1799, 11, 1803, pp. 512– 804.

47. P. Boyle, *The Publican and Spirit Dealers' Daily Companion, or Plain and Interesting Advice to Wine Vault and Public House Keepers*, 6th ed., London, 1808.

48. Douglas, *"Distilleries."*

49. P. Jonas, *The Genuine Art of Gauging Made Easy and Familiar*, London: C. Wittingham, 1806.

50. W. Speer, *An Enquiry into the Causes of the Errors and Irregularities which Take Place in Ascertaining the Strengths of Spirituous Liquors by the Hydrometer*, London: Payne and Mac'Kenlay, 1802.

51. Tate and Gabb, *Alcoholometry*.

52. Dring & Fage import and export hydrometers with hanging weights (vs. screwed), not of the centesimal design, were produced through the mid-19th century. They also did not place instrument readings on a continuous line.

53. J. Owens, *Plain Papers Relating to the Excise Branch of the Inland Revenue Department*, Linlithgow, 1879.

54. R. East, *Choice Chips of Revenue Lore*, Portsmouth: Hampshire Telegraph, 1877.

55. R. Clarke, *Notice is Hereby Given to all Dealers in Brandy, Rum, Malt, or Melasses-Spirits, Arrack, &c. That the Hydrometer or Brandy-Prover, Being the Production of many Years Study and Experiments, is Now Brought to its Utmost Perfection*, London, 1747.

56. B. Martin, *A Sure Guide to Distillers, and All Dealers in Spirituous Liquors, For Discovering the True Proportion of Water and Alcohol in Any Proposed Compound; and How to Make it Exactly Proof*, London, 1759.

57. Douglas, *"Distilleries."*

58. Blagden, *"Proportioning excise."*

59. B. Woodcroft, Alphabetical Index of Patentees of Inventions, From March 2, 1617 (14 James I) to October 1, 1852 (16 Victoriæ). London: Eyre and Spottiswoode, 1854.

60. #13 in https://osgalleries.org/collectors/wyman/wymanthumbnails.cgi.

61. Quin, *New Patent Hydrometer*.

62. A. D. Morrison-Low, *Making Scientific Instruments in the Industrial Revolution*, Burlington, Vermont: Ashgate, 2007.

63. *"Hydrostatics,"* Encyclopædia Britannica, Vol. 9, Dublin: James Moore, 1792.

64. Scarisbrick, *Spirit Assaying,* p. 198.

65. *"Description of Matthew Quin's improved hydrometer for spirits and worts,"* Transactions of the Society, Instituted at London, for the Encouragement of Arts, Manufactures, and Commerce, 8, 1790, pp. 198–203.

66. G. Clifton, *Directory of British Scientific Instrument Makers 1550–1851*, London: Zwemmer, 1995.

67. Fletcher, *"Atkin's hydrometer."*

68. Atkins & Co., Specific gravities, p. 31.

69. Atkins & Co., Specific gravities.

70. Jonas, *Gauging.*

71. Minutes of evidence taken by the Royal Commission on Whiskey and Other Potable Spirits. London: Jas Treuscott & Sons, 1908.

72. §6 of 42 & 43 Victoria c. 30, 1879, requires that brandy, whisky, and rum be 75°IP or stronger, and that gin be 65°IP or stronger.

73. "Federal Alcohol Administration Regulations No. 5," Federal Register 1:40, April 2, 1936, pp. 92-103.

74. Statutory Rules and Orders 1946 No. 2169. "Emergency Laws (Transitional Provisions) Food (Labelling)"

75. M. and T.M. Sheridan, *The Spirit Traders' and Licensed Victuallers' Reducing Table*, London, 1890.